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LETTER TO THE EDITOR

Zero beta function for a model of diffusion in potential random field

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Abstract. A rigorous perturbative proof is given of both the renormalisability of the field-theoretic model of diffusion in potential random field, and the triviality of the renormalisation group beta function.

We consider the following stochastic problem of a random walk [1, 2]:

$$\dot{x}_m = F_m(x) + \eta_m \qquad x_m = x_m(t) \qquad \eta_m = \eta_m(t)$$

with given drift (velocity) field $F_m(x)$ and Gaussian noise η with zero mean and correlation function $\overline{\eta_m(t)\eta_n(t')} = 2D_0\delta(t'-t)\delta_{mn}$, where D_0 is the diffusion coefficient (all parameters shall be renormalised, symbols with the subscript zero refer to their bare values, and symbols without this subscript to their renormalised values). The distribution function $P(x, t)$ for $t>0$ and arbitrary initial conditions satisfies the Fokker–Planck equation

$$[\partial_t + \partial_m(F_m - D_0\partial_m)]P \equiv LP = 0. \tag{1}$$

We are interested in the retarded Green function L^{-1} of this equation. It is convenient to exclude the variable t by a Fourier transformation:

$$G_\omega = L_\omega^{-1} \qquad L_\omega = -\mathrm{i}\omega + \nabla(F - D_0\nabla). \tag{2}$$

We express the drift field F_m as a sum of a fixed external part F_m^{ext} and a random part F_m^{st} with a Gaussian distribution with zero mean and the correlation function

$$\langle F_m^{\mathrm{st}}(x)F_n^{\mathrm{st}}(x')\rangle = \int \frac{\mathrm{d}k}{(2\pi)^d}(\lambda_0^\perp P_{mn}^\perp(k) + \lambda_0^\parallel P_{mn}^\parallel(k))\exp[\mathrm{i}k(x-x')] \tag{3}$$

where d is the space dimensionality and P are the projection operators:

$$P_{mn}^\perp(k) = \delta_{mn} - k_m k_n/k^2 \qquad P_{mn}^\parallel(k) = k_m k_n/k^2. \tag{4}$$

The quantities λ_0 in (3) are non-negative constants, which for $\lambda_0^\perp = \lambda_0^\parallel$ correspond to an isotropic correlation function, and for $\lambda_0^\parallel = 0$, $\lambda_0^\perp = 0$ to purely transverse and longitudinal correlation functions, respectively (models I, II, and III of [2]).

In the present letter we consider the case of purely longitudinal drift $\lambda_0^\perp = 0$. In this case the drift field $F_m(x)$ is proportional to the gradient of a 'potential' $\theta(x)$, and if we want to have for (1) a stationary solution

$$\rho(x) = \exp(-\theta(x)/T_0) \qquad \theta(x) = \theta^{\mathrm{ext}}(x) + \theta^{\mathrm{st}}(x) \tag{5}$$

with a given temperature T_0, we must set

$$F_m(x) = -D_0 \partial_m \theta(x)/T_0.$$

As $\theta^{\mathrm{ext}}(x)$ we choose a linear function of the coordinate, setting

$$\theta^{\mathrm{ext}}(x)/T_0 = E_{0m} x_m \qquad \theta^{\mathrm{st}}(x)/T_0 = \psi(x)$$

where the fixed arbitrary vector E_{0m} determines the external (non-random) drift $F_m^{\mathrm{ext}} = -D_0 E_{0m}$, and $\psi(x)$ the random component of the drift $F_m^{\mathrm{st}} = -D_0 \partial_m \psi$. For the Fourier transform of the correlation function $D_{\psi 0}(x, x') = \langle \psi(x) \psi(x') \rangle$ in the longitudinal case $\lambda_0^\perp = 0$ we obtain from (3)

$$D_{\psi 0}(k) = g_0/k^2 \qquad g_0 = \lambda_0^\parallel / D_0^2. \tag{6}$$

To average the Green function (2) G_ω over the random field ψ with the corresponding weight

$$\exp\left(-\frac{1}{2}\psi D_{\psi 0}^{-1} \psi\right) = \exp\left(\frac{1}{2g_0}\psi \nabla^2 \psi\right) \tag{7}$$

(integral over x in the exponent here and in other analogous formulae is implied), we use the functional integral representation:

$$G_\omega(x, x') = \det L_\omega \int D\tilde{\varphi} D\varphi \varphi(x) \tilde{\varphi}(x') \exp(-\tilde{\varphi} L_\omega \varphi). \tag{8}$$

Equations (7) and (8) lead to a field theory with the action

$$S = -\tfrac{1}{2}\psi D_{\psi 0}^{-1} \psi + \tilde{\varphi}[m_0 + \nabla(\nabla + E_0 + \nabla\psi)]\varphi \tag{9}$$

where we have scaled the fields φ and $\tilde{\varphi}$ so that $m_0 = \mathrm{i}\omega/D_0$, and omitted the term $\mathrm{Tr} \ln L_\omega$. The only effect of this term is to cancel graphs with closed loops of the $\varphi\tilde{\varphi}$ propagator, and we shall neglect the contribution of such graphs by convention. The averaged Green function $\langle G_\omega \rangle$ is simply related to the full $\varphi\tilde{\varphi}$ propagator G of the field theory (9): $D_0 \langle G_\omega \rangle = G$.

Canonical dimensions d_A of the quantities A in the action (9) are

$$d_\varphi + d_{\tilde{\varphi}} = d - 2 \qquad d_\psi = 0 \qquad d_{m_0} = 2 \qquad d_{E_0} = 1 \qquad d_{g_0} = 2 - d.$$

Thus, the model is logarithmic (i.e. the bare coupling constant g_0 is dimensionless) at $d = 2$, and we shall introduce dimensional regularisation by the shift $d = 2 - \varepsilon$. Ultraviolet (UV) divergences of the logarithmic theory appear in the form of poles in ε and they are removed in the standard fashion with the help of necessary counterterms. These are added not to the action (9), but to the basic action:

$$S_{\mathrm{B}} = -\tfrac{1}{2}\psi D_\psi^{-1} \psi + \tilde{\varphi}[m + \nabla(\nabla + E + \nabla\psi)]\varphi \tag{10}$$

where

$$D_\psi(k) = g\mu^\varepsilon / k^2 \tag{11}$$

and μ is the scaling parameter ($d_\mu = 1$), and m, E, g are the renormalised parameters of the model. The renormalised coupling constant g is dimensionless: $d_g = 0$, whereas $d_m = 2$ and $d_E = 1$.

The counterterms are determined from the structure of interaction

$$S_{\mathrm{int}} = \tilde{\varphi}\nabla(\nabla\psi\varphi) = -\nabla\tilde{\varphi}\nabla\psi\varphi \tag{12}$$

(integration by parts has been used in the implicit integral over x) and from the dimensions of one-particle-irreducible (1PI) Green functions of the basic theory at $\varepsilon = 0$. All the fields $\tilde{\varphi}$, φ, ψ are dimensionless at $d = 2$, and therefore the canonical dimension of any 1PI Green function in the momentum space (the 'formal' UV index) is equal to 2, while the 'real' UV index δ' [3] is given by

$$\delta' = 2' - N_{\tilde{\varphi}} - N_\psi \tag{13}$$

where $N_{\tilde{\varphi}}$ and N_ψ are the numbers of external legs of the fields $\tilde{\varphi}$, ψ in the 1PI Green function. From (12) it is obvious that for each external leg of $\tilde{\varphi}$ and ψ in a given 1PI graph external momenta corresponding to the derivatives in (12) factorise, thus lowering the dimension of the remaining loop integral. From (12) and (13) it follows that in the generic case the counterterms

$$\tilde{\varphi}\nabla^2\varphi \qquad \tilde{\varphi}\nabla(\boldsymbol{E}\varphi) \qquad \tilde{\varphi}\nabla(\nabla\psi\varphi) \qquad \varphi\nabla\tilde{\varphi}\varphi\nabla\tilde{\varphi} \tag{14}$$

should be added to the action of the basic theory in order to remove divergences (poles in $\varepsilon = 2 - d$). The counterterm $\psi\nabla^2\psi$ is also permitted by dimensional considerations. However, the propagator $\psi\psi$ is not renormalised at all due to absence of graphs with loops of the $\varphi\tilde{\varphi}$ propagator. The first three counterterms in (14) have the same structure as terms in the initial action (9). Provided the fourth counterterm in (14) is absent, addition of the first three to the basic action would lead to a renormalised action of the form

$$S_{\mathrm{R}} = -\tfrac{1}{2}\psi D_\psi^{-1}\psi + \tilde{\varphi}[m + \nabla(Z_1\nabla + Z_2\boldsymbol{E} + Z_3\nabla\psi)]\varphi \tag{15}$$

where Z_i are the renormalisation constants (series in g and $1/\varepsilon$). The functionals (9) and (15) are related in the standard way of multiplicatively renormalised theory:

$$S_{\mathrm{R}}(\varphi, \tilde{\varphi}, \psi) = S(Z_\varphi\varphi, Z_\varphi\tilde{\varphi}, Z_\psi\psi) \tag{16}$$

and

$$m_0 = mZ_m \qquad \boldsymbol{E}_0 = \boldsymbol{E}Z_E \qquad g_0 = g\mu^\varepsilon Z_g. \tag{17}$$

From (15)-(17) it follows

$$Z_m Z_\varphi^2 = Z_\psi^2 Z_g^{-1} = 1 \qquad Z_1 = Z_\varphi^2 \qquad Z_2 = Z_\varphi^2 Z_E \qquad Z_3 = Z_\varphi^2 Z_\psi. \tag{18}$$

Now we may explicitly formulate the main conjectures of this paper: (i) $Z_2 = Z_1$, (ii) $Z_3 = Z_2$ and (iii) the last counterterm in (14) is indeed absent, i.e. the model is multiplicatively renormalisable. From (i), (ii) and (18) we obtain

$$Z_E = Z_\psi = Z_g = 1$$

and $Z_g = 1$ renders the renormalisation group β function trivial: $\beta(g) = -\varepsilon g$.

The relation $Z_2 = Z_3$ may be obtained by a simple graphical analysis [4] but conjectures (i) and (iii) are not trivial. They have been checked to two-loop order in [2]. The authors of [4] claim to have given a general proof in the following way: in formula (5) the factor $\exp(-\theta^{\mathrm{ext}}(\boldsymbol{x})/T_0) = \exp(-\psi^{\mathrm{ext}}(\boldsymbol{x}))$ obviously is not affected by averaging over $\psi = \theta^{st}/T_0$. Thus, according to the authors of [4], the quantity $\psi^{ext}(x)$ is not renormalised, which in the case of $\psi^{ext}(x) = E_{0m}x_m$ would mean that the parameter E is not renormalised, i.e. $Z_E = 1 \Leftrightarrow Z_1 = Z_2$. However, this is actually not a proof, since it is not the quantity (5) which is averaged over the random field, but the Green function (8), from which the contribution of ψ^{ext} cannot be extracted (contrary to (5)) as a multiplier. If the dependence of G_ω on ψ^{ext} has been simple, then the proof of the required relation $Z_1 = Z_2$ would, of course, have become trivial. However, this is not the case; therefore the authors of [4], having put forward a conjecture, failed to prove it.

A different recent approach [5] to this problem is based on a connection of the action (15) to a non-linear σ model. However, this treatment involves a non-linear transformation of fields; therefore the properties of the σ model cannot be directly transferred to the initial diffusion model. Since the analysis of this relation has not been presented, we consider it an open question, whether or not this approach leads to an alternative correct proof of the conjectures (i), (ii) and (iii).

The contributions to the counterterms (14) are extracted from the following 1PI Green functions of the basic theory: $\Gamma_{\tilde{\varphi}\varphi}$, $\Gamma_{\tilde{\varphi}\varphi\psi}$ and $\Gamma_{\tilde{\varphi}\tilde{\varphi}\varphi\varphi}$. By definition, $\Gamma_{\tilde{\varphi}\varphi} = -G^{-1}$, where G is the full $\varphi\tilde{\varphi}$ propagator. In all graphs corresponding to these 1PI Green functions, momenta flowing to external $\tilde{\varphi}$ and ψ legs may be factorised, and the remaining loop integrals are either linearly ($\Gamma_{\tilde{\varphi}\varphi}$) or logarithmically ($\Gamma_{\tilde{\varphi}\varphi\psi}, \Gamma_{\tilde{\varphi}\tilde{\varphi}\varphi\varphi}$) divergent in the logarithmic theory ($d = 2$). Therefore the primitive divergences (i.e. divergences remaining after subtraction of the contributions of divergent subgraphs) of these Green functions have the following structures:

$$KR'\Gamma_{\tilde{\varphi}\varphi}(p) = -p(C_1 p - C_2 \mathrm{i}E) \tag{19}$$

$$KR'\Gamma_{\tilde{\varphi}\varphi\psi}(q, -q-k, k) = C_3 qk \tag{20}$$

$$KR'\Gamma_{\tilde{\varphi}\tilde{\varphi}\varphi\varphi}(q, q', p, -p-q-q') = C_4 qq' \tag{21}$$

where $KR'\Gamma$ denotes the primitively divergent part of the diagrammatic representation of the function Γ, and the external momenta flowing into the graphs are indicated in the order of the field arguments. C_i are dimensionless coefficients, which diverge in the limit $\varepsilon \to 0$ (series in g with polynomial in $1/\varepsilon$ coefficients in the standard scheme of minimal subtractions). We want to prove that to arbitrary order in the perturbation theory the following relations hold:

$$C_2 = C_1 \qquad C_3 = C_2 \qquad C_4 = 0. \tag{22}$$

These three equations correspond to our three main conjectures. Since C_i do not depend on the parameter m (as well as on any other dimensional parameter), we henceforth set $m = 0$.

To prove relations (22), we introduce the change of variables

$$\varphi(x) \to \varphi(x)\exp(-\psi(x)) \qquad \tilde{\varphi}(x) \to \tilde{\varphi}(x)\exp(\psi(x))$$

which leads to a field theory with the basic action ($m = 0$)

$$S_B = -\tfrac{1}{2}\psi D_\psi^{-1}\psi + \tilde{\varphi}(\nabla - \nabla\psi)(\nabla + E)\varphi \tag{23}$$

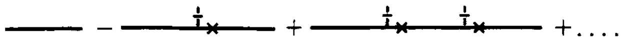
and for the full propagator of the initial model (10) we obtain the expression

$$G(x - x') = \langle \exp(-\psi(x))T(x, x')\exp(\psi(x'))\rangle \tag{24}$$

where angular brackets denote the average over ψ, and the quantity $T(x, x')$ is defined as

$$T(x, x') = G_0(x - x') - G_0[\nabla\psi(\nabla + E)G_0](x, x') + G_0[\nabla\psi(\nabla + E)G_0]^2(x, x') + \dots \tag{25}$$

or, graphically, as

$T =$  (26)

Here, full lines denote the bare propagator G_0 of the basic theory

$$G_0(x - x') = \int \frac{\mathrm{d}p}{(2\pi)^d} \frac{\exp[\mathrm{i}p(x - x')]}{p(p - \mathrm{i}E)} \tag{27}$$

crosses denote the operator $\nabla + E$, truncated broken lines denote ψ fields and the slash on them corresponds to the derivative at the interaction vertex of the basic action (23). The crucial property of expression (25) is that, apart from the common factor G_0 on the left, G_0 appears only in the combination

$$(\nabla + E)G_0(x - x') = \mathrm{i} \int \frac{\mathrm{d}p}{(2\pi)^d} \frac{p - \mathrm{i}E}{p(p - \mathrm{i}E)} \exp[\mathrm{i}p(x - x')]$$

where the quantity $(p - \mathrm{i}E)/p(p - \mathrm{i}E)$ does not diverge in the limit $p \to \mathrm{i}E$. The effect of the exponential factors in (24) may be expressed in the form of 'external' vertices with an arbitrary number of ψ fields, which are attached to both ends of the G_0 chains in the expansion (26). The the average over ψ graphically amounts to connecting both the 'normal' vertices in (26) and the 'external' ones by broken lines corresponding to the propagator $D_\psi(k) = g\mu^\varepsilon/k^2$. In graphs, where the leftmost propagator $G_0(p) = 1/p(p - \mathrm{i}E)$ is included in a loop of full and broken lines, the loop integral smears the pole at $p = \mathrm{i}E$. The pole survives only in graphs, where this propagator is left outside of all the loops. Thus, the Fourier transform of $G(x - x')$ may be expressed in the form

$$G(p) = \frac{R(p)}{p(p - \mathrm{i}E)} + \tilde{G}(p) \tag{28}$$

where R and $\tilde{G}$ do not diverge in the limit $p \to \mathrm{i}E$. Due to the relation $G = -\Gamma^{-1}_{\tilde{\varphi}\varphi}$ this means that

$$\Gamma_{\tilde{\varphi}\varphi}|_{m=0, p=\mathrm{i}E} = 0. \tag{29}$$

This equation holds also for the primitively divergent part of $\Gamma_{\tilde{\varphi}\varphi}$, and taking into account (19) we obtain $C_1 = C_2$. The relation (29) is interesting also from a slightly more general point of view. In field theory, the set of solutions φ_0 of the free equation $G_0^{-1}\varphi_0 = 0$ is usually called the 'mass shell'. For the propagator (27) the equation $p(p - \mathrm{i}E)\varphi_0(p) = 0$ has a solution $\varphi_0(p) \propto \delta(p - \mathrm{i}E)$, which in the coordinate space corresponds to the stationary Boltzmann distribution $\varphi_0(x) \propto \exp(-Ex)$ in a given external potential $\theta^{\mathrm{ext}}(x) = T_0(Ex)$. Equation (29) means that interaction with the random field ψ does not shift the mass shell. This is a non-trivial property of this particular model, since in general interaction always results in a shift of the mass shell (mass renormalisation in field theories).

For the full three-point function $G_{\varphi\tilde{\varphi}\psi}$ we obtain the expression

$$G_{\varphi\tilde{\varphi}\psi}(x, x', y) = \langle \exp(-\psi(x))\, T(x, x')\, \exp(\psi(x'))\, \psi(y) \rangle$$

and the properties of T allow us to present the Fourier transform of $G_{\varphi\tilde{\varphi}\psi}$ as

$$G_{\varphi\tilde{\varphi}\psi}(\boldsymbol{p}-\boldsymbol{k}, -\boldsymbol{p}, \boldsymbol{k}) = -\frac{R(\boldsymbol{p})D_{\psi}(\boldsymbol{k})}{p(p-\mathrm{i}E)} + \tilde{G}_3(\boldsymbol{p}-\boldsymbol{k}, -\boldsymbol{p}, \boldsymbol{k}) \tag{30}$$

where $\tilde{G}_3$ does not diverge in the limit $p \to \mathrm{i}E$. Using the connection

$$\Gamma_{\tilde{\varphi}\varphi\psi}(\boldsymbol{p}-\boldsymbol{k}, -\boldsymbol{p}, \boldsymbol{k}) = -\Gamma_{\tilde{\varphi}\varphi}(\boldsymbol{p}-\boldsymbol{k})G_{\varphi\tilde{\varphi}\psi}(\boldsymbol{p}-\boldsymbol{k}, -\boldsymbol{p}, \boldsymbol{k})\Gamma_{\tilde{\varphi}\varphi}(\boldsymbol{p})\Gamma_{\psi\psi}(\boldsymbol{k})$$

(where $\Gamma_{\psi\psi} = -D_{\psi}^{-1}$ due to the absence of fluctuation corrections) we obtain from (28) and (30) in the limit $\boldsymbol{p} \to \mathrm{i}\boldsymbol{E}$

$$\Gamma_{\tilde{\varphi}\varphi\psi}(-\boldsymbol{k}+\mathrm{i}\boldsymbol{E}, -\mathrm{i}\boldsymbol{E}, \boldsymbol{k}) = \Gamma_{\tilde{\varphi}\varphi}(-\boldsymbol{k}+\mathrm{i}\boldsymbol{E}) \tag{31}$$

which, with the use of (19), (20) and (29) leads to the relation $C_2 = C_3$.

The full four-point function $g_{\varphi\varphi\tilde{\varphi}\tilde{\varphi}}$ may be expressed as

$$G_{\varphi\varphi\tilde{\varphi}\tilde{\varphi}}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x}', \boldsymbol{y}') = \langle \exp(-\psi(\boldsymbol{x}) - \psi(\boldsymbol{y}))$$
$$\times (T(\boldsymbol{x}, \boldsymbol{x}')T(\boldsymbol{y}, \boldsymbol{y}') + T(\boldsymbol{x}, \boldsymbol{y}')T(\boldsymbol{y}, \boldsymbol{x}')) \exp(\psi(\boldsymbol{x}') + \psi(\boldsymbol{y}'))\rangle.$$

Analogously, we can extract the most singular part of the Fourier transform of $G_{\varphi\varphi\tilde{\varphi}\tilde{\varphi}}$ in the limit $\boldsymbol{p}, \boldsymbol{p}' \to \mathrm{i}\boldsymbol{E}$, where $\boldsymbol{p}$ and $\boldsymbol{p}'$ are the momenta, flowing from the external $\tilde{\varphi}$ legs of the corresponding graphs. This part is given by graphs, where the broken lines connect the left ends of the two T lines with each other only (but not with any other vertices). Therefore

$$G_{\varphi\varphi\tilde{\varphi}\tilde{\varphi}}(\boldsymbol{q}, \boldsymbol{p}+\boldsymbol{p}'-\boldsymbol{q}, -\boldsymbol{p}, -\boldsymbol{p}') = \frac{2R(\boldsymbol{p})R(\boldsymbol{p}')B(\boldsymbol{q}-\boldsymbol{p})}{p(p-\mathrm{i}E)p'(p'-\mathrm{i}E)} + \dots \tag{32}$$

where ... denotes less singular terms, and the quantity B is defined by a series of ψ propagators (11) as follows:

$$B(\boldsymbol{k}) = \int \mathrm{d}\boldsymbol{x}\, \exp(-\mathrm{i}\boldsymbol{k}\boldsymbol{x})\left(D_{\psi}(\boldsymbol{x}) + \frac{1}{2}D_{\psi}^2(\boldsymbol{x}) + \frac{1}{3!}D_{\psi}^3(\boldsymbol{x}) + \dots\right).$$

Using again the relation between full and 1PI Green functions we obtain from (28), (31) and (32)

$$\Gamma_{\tilde{\varphi}\tilde{\varphi}\varphi\varphi}(\boldsymbol{k}+\mathrm{i}\boldsymbol{E}, -\boldsymbol{k}+\mathrm{i}\boldsymbol{E}, -\mathrm{i}\boldsymbol{E}, -\mathrm{i}\boldsymbol{E}) = 2\Gamma_{\tilde{\varphi}\varphi}(\boldsymbol{k}+\mathrm{i}\boldsymbol{E})[B(\boldsymbol{k}) - D_{\psi}(\boldsymbol{k})]\Gamma_{\tilde{\varphi}\varphi}(-\boldsymbol{k}+\mathrm{i}\boldsymbol{E}).$$

This relation shows that the four-point function does not contain terms with the required local structure (21), which means that there are no promitive divergences and $C_4 = 0$.

Finally, we show that our proof applies also to the model of diffusion with 'long-range' correlated random drift [5, 6]. In this case, the correlation function of the random field $\boldsymbol{F}^{\mathrm{st}}$ is of the form

$$\langle F_m^{\mathrm{st}}(\boldsymbol{x})F_n^{\mathrm{st}}(\boldsymbol{x}')\rangle = \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^d}(\lambda_0^{\perp}P_{mn}^{\perp}(\boldsymbol{k}) + \lambda_0^{\parallel}P_{mn}^{\parallel}(\boldsymbol{k}))\frac{\exp[\mathrm{i}\boldsymbol{k}(\boldsymbol{x}-\boldsymbol{x}')]}{k^{2\alpha}}$$

where $\alpha > 0$ is a new parameter, and projection operators P are defined by (4). In the potential case, the correlation function (6) $D_{\psi 0}$ is replaced by

$$D_{\psi 0}(\boldsymbol{k}) = g_0/k^{2\alpha}.$$

Due to this, the corresponding field theory is logarithmic at $d = 2 + 2\alpha$ dimensions. However, there are no other changes in the model, and for our argument the explicit form of the ψ propagator is not essential. Therefore, our proof remains valid also for 'long-range' correlated potential drift. In fact, the conjecture (iii) becomes trivial in this case, since the real UV index of 1PI Green functions is given by

$$\delta' = 2 + 2\alpha - N_{\tilde{\varphi}}(1 + 2\alpha) - N_{\psi}$$

which is negative for $N_{\tilde{\varphi}} = 2$, $N_{\psi} = 0$. Thus, the four-point term (14) may be excluded by dimensional arguments.

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